Conceptual Understanding of Undergraduate Students of Calculus in Cooperative Learning Using Calculus Education Software (CES)

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Abstract

The aim of this study is to enhance conceptual understanding of undergraduate students in the Calculus, a vital branch of mathematics and to find out how students may use different kinds of representations for thinking about the concepts in Calculus. To achieve this, I designed a three level teaching experiment to study the effects of computer assisted interactive teaching of calculus course at undergraduate students. At level one, students' conceptions/misconceptions about calculus especially function concept, ε-δ definition of limit of a function, continuity/discontinuity of function, derivability of function in an interval and at the endpoints were investigated through the activities like diagnostic test (pre-test). At the second level, Calculus Education Software (CES) was developed to get rid of misconceptions and to improve conceptual understanding of the students. After pre-test, three groups of students; control group, experimental group and cooperative group were formed. All the three groups were demonstrated the same topics of calculus using different teaching methods and learning effect was calculated using t-test by comparing mean scores of posttest administered after the treatment.

Introduction:

A number of efforts have been made by mathematics educators in building calculus concepts easier, interesting and motivating without any harm to natural phenomena of the subject. Calculus teaching at undergraduate level had been focused on students' learning strategies, without essentially giving attention to intuition and to the creation of several representations of concepts which contribute to their considerable understanding. Several studies have revealed the positive effects of cooperative and interactive learning, including one to one dialog of teacher with student as well student with student, student's willingness to answer and ask questions, increased level of confidence of students, comprehensive and correct understanding, increased conceptual understanding, and increased ability to apply knowledge in solving problems [10, 15, 17].

In developing calculus concepts in correct and lucid manner, the preconceptions about the topic that fixed in the mental makeup of students that play an important role should be considered. These preconceptions might have been perforated either from teachers who taught them previous courses or the use of the mathematical term of the concept in day to day life. For example, students may have had experiences in everyday life where the word limit is involved as in such cases as speed limit, capacity. Such everyday language connotations may therefore interfere with students' understanding of the mathematical notion of limits. The correct intuitions and the understanding of a concept are taken to evolve through the creation of multiple representations of the concept. It is important that the teaching that uses multiple representations of the concept such as graphical and algebraic might help students to learn and understand with the correct intuitions. Multiple perspectives of the concepts can easily be elaborated by means of computer software or simulations.

Review of literature:

It is widely accepted that calculus concepts are abstract and complex for students to understand. Teaching and learning of these concepts may be challenging and even exasperating at times [8]. Students should construct mathematical knowledge by solving problems and not just memorizing procedures, by investigating patterns and not just memorizing formulas, and by forming conjectures and not just doing exercises [9]. This suggests that multi-dimensional approach should be emphasized in teaching the abstract calculus concepts for conceptual understanding.

Computers in Mathematics Education:

Computers have promoted entirely new fields in the era of mathematics education providing innovative visual ways to represent mathematical information. Computer based instructions in mathematics mainly focus on drills, practice and tutorials which intensify mathematical abilities of learners and stimulates for mathematical thoughts. Research suggests that despite the numerous benefits of using technology in mathematics education, the process of embedding technology in classrooms is slow and complex [3]. Computer algebra systems (such as Derive, Mathematica, Maple or MuPAD) and dynamic geometry software (such as Geometer's Sketchpad or Cabri Geometry) are powerful technological tools for teaching mathematics. Numerous research results showed that these software packages may be used to encourage discovery and experimentation in classrooms and their visualization features may be effectively employed in teaching to generate conjectures [13].

Students' Understanding of Calculus:

Traditional calculus courses tend to focus more on algebraic drill and practice on calculus problems without understanding the underlying concepts. The calculus curriculum should be reformed by putting more emphasis on conceptual understanding of the fundamentals of calculus and complementing the use of graphical, numerical, algebraic and verbal representation in the teaching and learning of calculus. Students' reluctance to visual concepts in calculus was reported by giving examples in which visual representations would solve certain problems almost trivially. Yet it was observed that students refrain from using them because the preference developed over the years is for a numerical, symbolic mode of approach [5, 6]. Algebraic manipulation is the preferred mode of operation for many students. However research shows that visual images may provide vital insights.

The understanding of functions does not appear to be easy, given the diversity of representations related to the concept [12]. Students have difficulties in making the connections between different representations of the notion (formulas, graphs, diagrams, and word descriptions), in interpreting graphs and manipulating symbols related to functions [14]. Researchers had revealed that students who have a coherent understanding of the concept of functions (geometric approach) may easily understand the relationships between symbolic and graphic representations in problems and are able to provide successful solutions. Moreover, it was observed that there is a close relationship between the use of a geometric approach in functions and better understanding of equations, graphs and functions in general [1].

The derivative concept was being explained as a rate of change in one quantity with respect to another quantity. However, presently many students are taught in a way that enables them to solve calculus problems without attending to rates of change [2]. It was observed that students memorized properties of second derivatives but could not relate it while discussing inflection points of the function graphically [2]. Tall (1986) and Ubuz (2007) reported students' difficulties in creating graphical representations of function's rate of change. These researchers found that students often focused on computing derivatives without connecting the derivatives they computed and evaluated to a function's rate of change at specific points in its domain [16, 18].

Research Objectives:

Objective of this study was to investigate students' difficulties in understanding calculus. In order to overcome through the observed difficulties; develop Calculus Education Software (CES) that may enhance conceptual understanding of topics in Calculus.

Methodology:

Since, experimental research provides a systematic and logical method for answering the question such as "Is there a difference in performance between participants who receive treatment A and participants who receive treatment B?" [7]. Here, in this study experimental research design is used.

Sample:

Sample of 60 first year undergraduate students offering mathematics as one of the subjects were selected by simple random sampling from the three colleges affiliated to Pune University. Pune University is well known as a one of the leading university of India. Three colleges from different parts of the city were selected randomly which were easily approachable and ready to provide help for smooth conduction of the experiment. Following, Table 1 shows the colleges and the number of students that were part of the study.

TABLE 1

Participants of the study (Year 2010-11)

College	Participants		
	Male	Female	Total
Annasaheb Magar College, Hadapsar, Pune	08	12	20
Baburaoji Gholap College, Sangvi, Pune	12	08	20
Prof. Ramkrishna More College, Akurdi, Pune	11	09	20

Total	**31**	**29**	**60**

Instrument:

It is essential in any experimental research, to design proper instrument for determining the approximate state of the students' knowledge about the subject. For this purpose diagnostic test (instrument) was administered to the participants. Instrument was containing multiple choice questions (items) on particular topics of calculus especially function concept, limit, continuity and differentiability of a function of a single variable. This instrument which was further considered as the pre-test of the experiment was validated by senior teachers of Mathematics having more than ten years teaching experience. Students had to choose correct alternative for each item with suitable justification.

After the pre-test analysis of students' responses, average difficulty index was calculated to measure the difficulty level of a diagnostic test and found to be 0.44 proving that the test is moderately difficult. Average discrimination index, a measure of the discriminatory power of diagnostic test i.e. the extent to which the diagnostic test distinguishes students who have good understanding of the subject from those who do not, is calculated as 0.34 showing test is good discriminator. To find whether the test is reliable or not, the reliability index of the test has been determined according to Kuder-Richardson method and found to be 0.713 which is acceptable for group measurement [4].

Treatment:

To investigate difficulties in the understanding calculus concept, pre-test was administered to the participants selected for the study. Calculus Education Software (CES) was developed by researcher to accomplish the needs of the conceptual understanding. After pre-test selected sample of 60 students of the first year undergraduate class was divided into three groups, namely control, experimental and cooperative groups. Students of control group were taught the topic by traditional teaching method while for experimental group traditional teaching was supported with the demonstrations using Calculus education software. For both of above groups students had passive role of observing blackboard or CES demonstrations.

Like experimental group, the third cooperative group was also taught with the support of CES initially. Then subgroups of cooperative group, each of three students were formed. One computer was provided to each subgroup and allowed to operate CES under guidance of teacher. Here students were in the mode of active learners. After the treatment, the post-test was administered to three groups of students to measure effects of CES on students' conceptual understanding of calculus.

Computer Education Software

After identifying student difficulties in learning calculus, researcher developed Calculus Education Software (CES) in C programming language on topics including Function concept, Limit, continuity, differentiability, maxima-minima of a function, mean value theorems. Programming language knowledge is not needed to the instructor as well as students to operate the software. Since it is converted to executable file, it runs on double clicking on any computer having windows operating system. No need of installing turbo C compiler. Here the visualization property of software mainly focuses on graphical illustration of the definition of function as well as graphical aspects of injectivity, surjectivity, bijectivity and invertability of a function. Different graphs of standard functions can be drawn and observed the differences which help students in creating mental images about nature of functions. Here user has an option of choosing range for *x*, different parameters and so on.

Limit and continuity part initially focuses on graphical illustration of deleted neighborhood of a point, ε-δ definition of a limit of a function. It also focuses on how values of δ changes on different choices of ε values in the ε-δ definition. It also demonstrates graphically a number of examples with existence or nonexistence of left hand and right hand limits.

In continuity part, ε-δ definition of continuity of a function is discussed by covering the cases of functions having limit at a point but not continuous thereof. Removable, irremovable discontinuities are interpreted graphically.

In differentiability, differentiability of function at a point, in the interval [a, b], at endpoints of the interval is discussed. How differentiability of a function graphically reflects in discussing the increasing/decreasing nature of function, in finding the points of inflection and maxima-minima of function are interpreted. Mean value theorems especially Rolle's and Lagrange's theorem are illustrated graphically. Illustration of differentiability implies continuity is discussed.

1. Graphical illustration of ε-δ definition of a limit of a function

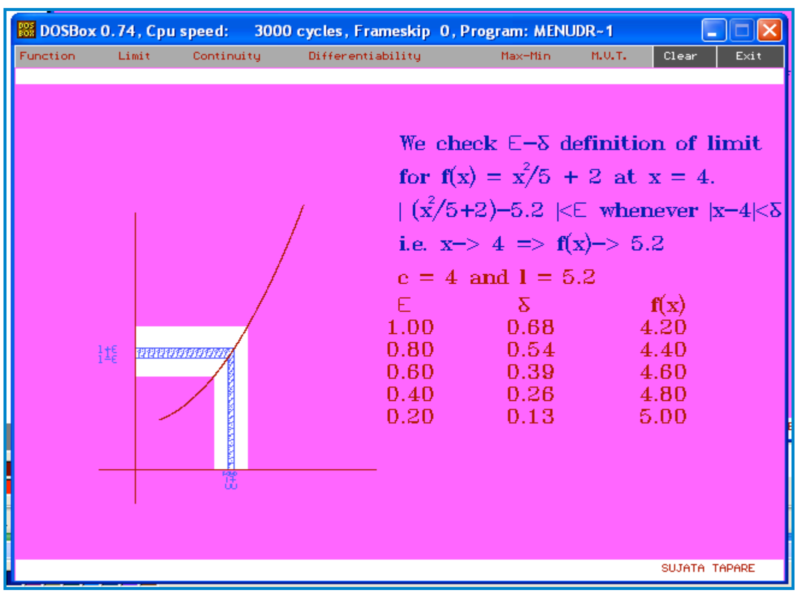


Fig. 1 Illustration of existence of limit using ε-δ def. of limit at point *x* = 4

Here illustration of ε-δ definition of limit is shown for $f(x) = \overline{} + 2 \;\; \forall\, x \in [1, 7]$ at $x = 4$.

Off course limit l is 5.2. To show that the limit value of $f(x) \rightarrow 5.2$, as $x \rightarrow 4$, the gradual decrease of ε is demonstrated through animated projection of the interval (l–ε, l+ε) on the curve and at each step corresponding δ-nhd of c i.e. (c–δ, c–δ) is obtained. Students able to visualize that for different ε value, how δ changes and $f(x)$ approaches to l.

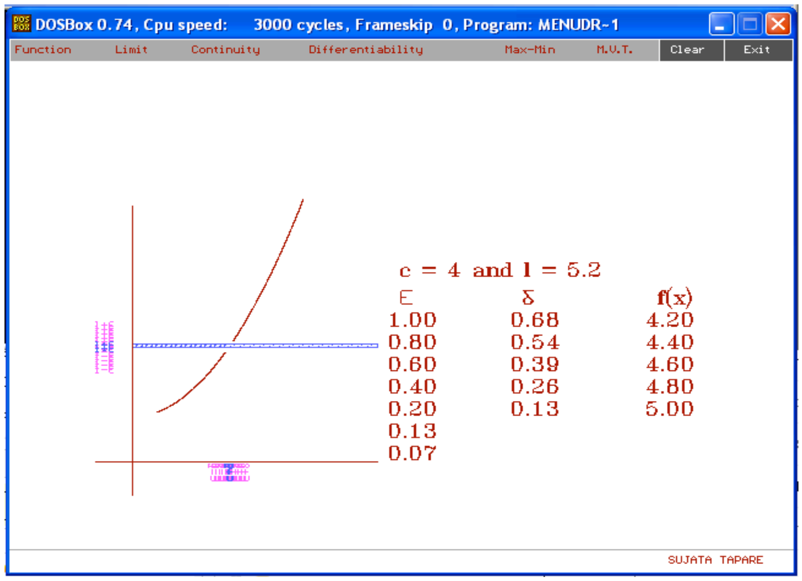


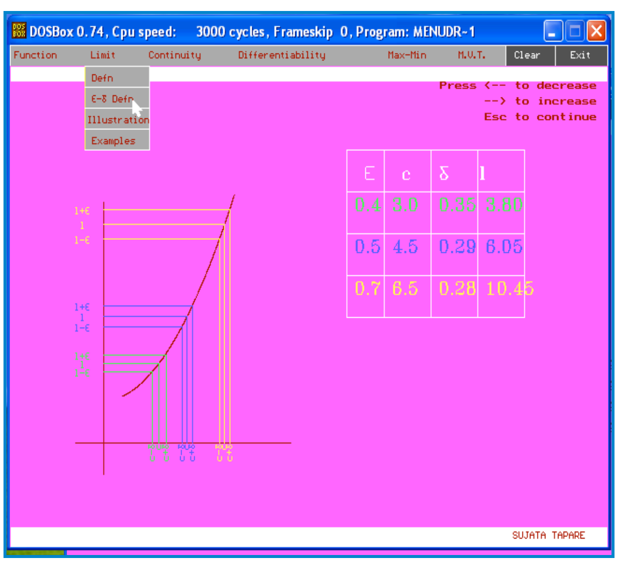
Fig. 2 Illustration of non-existence of limit using ε-δ def. of limit at point $x = 4$

In this animation, limit of $f(x)$ does not exist at $x = 4$ is shown for

$$f(x) = - + 2 \quad \forall\ x \in [1,3.8] \cup [4.1,7].$$

For some values of ε (ε = 1.0, 80, 0.60, 0.40, 0.20) there exists δ satisfying ε-δ definition of limit. But as value of ε decreases it can be observed that δ value cannot be obtained. Thus non-existence of limit is graphically revealed by visualization of non-existence of interval (c–δ, c–δ) corresponding to the *each* interval (*l*–ε, *l*+ε), *for any* ε > 0.

2. Some other screenshot of CES



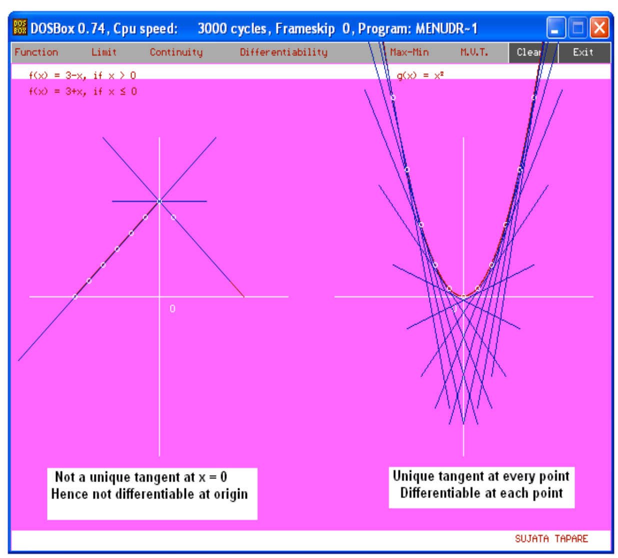


Fig. 3 ε-δ def. of limit at multiple points **Fig. 4 Illustration of Differentiability**

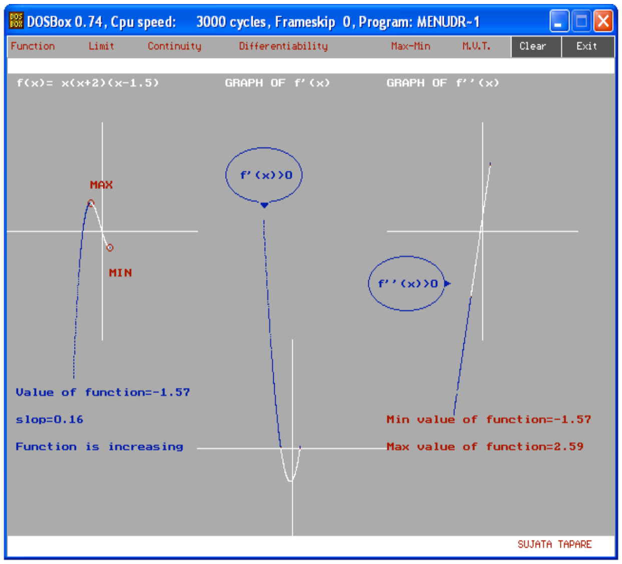


Fig. 4 Illustration of Maxima-minima

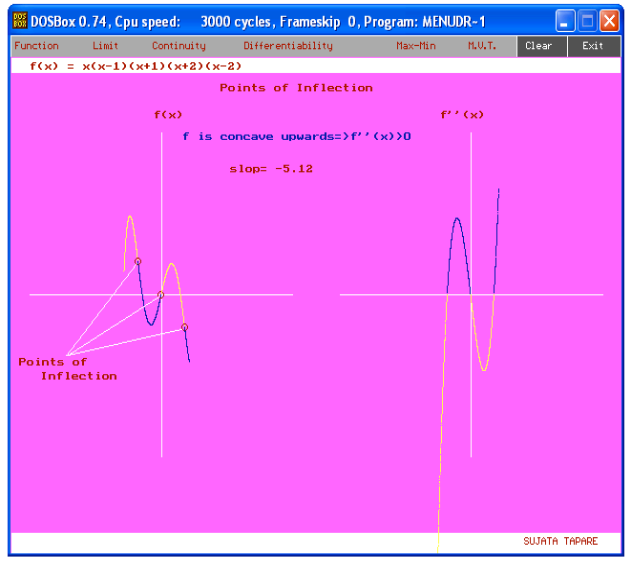


Fig. 5 Illustration of Points of Inflection

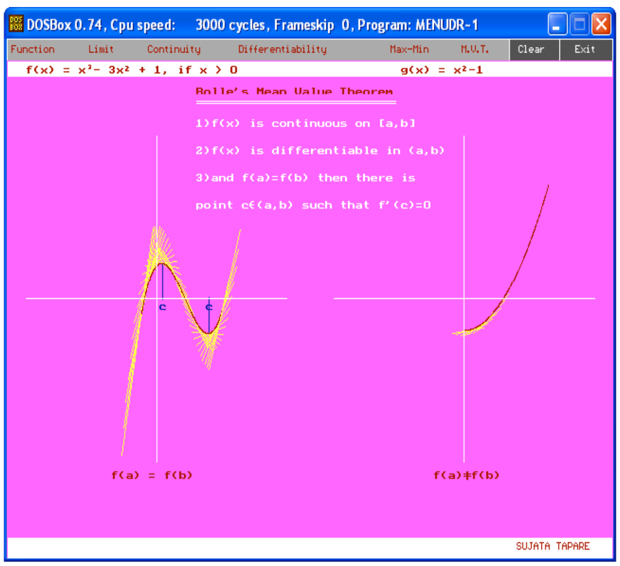


Fig. 6 Illustration of Rolle's MVT

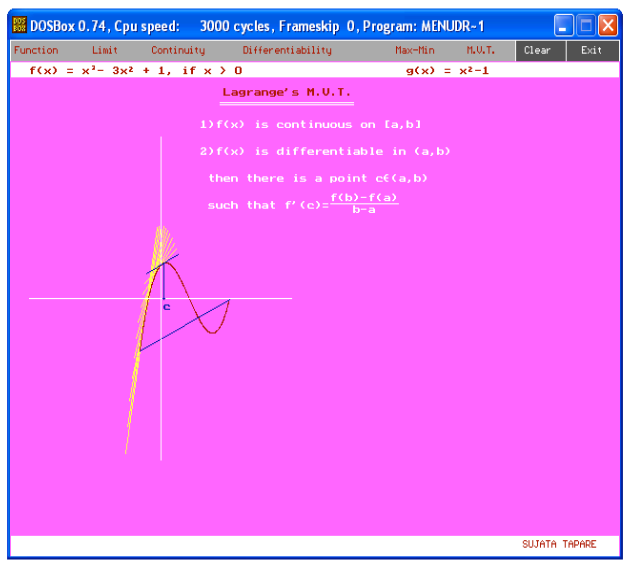


Fig. 7 Illustration of Lagrange's MVT

Data Analysis and Results:

To compare the pre-test and post-test mean scores, the normalized gain method was used. The normalized gain is independent of the pre-test scores that lead us to expect that if a diverse set of classes has a wide range of pretest scores but all other learning conditions are similar, the value of normalized learning gain measured in the different classes would not differ significantly. This independence of normalized gain also suggests that a measurement of difference in <g> between two groups having very different pre-test scores would be reproduced even by a somewhat different test instrument which results in a shifting of pre-test scores. Average normalized gain <g> is a much better indicator of the extent to which a treatment is effective than is either gain or post-test. If the treatment yields <g> > 0.3 for a course, then the course could be considered as in the “interactive-engagement zone.” [11]

Initially, using pre-test and post-test scores of each student the normalized gain ‘g’ of each student of the experimental group and the control group was obtained by using formula,

g = ______________

Then average normalized gain <g> for each group with its standard deviation was obtained for each group and results are summarized as below in the Table 2.

TABLE 2

Comparison of pre-test and post-test scores

		Group		
		Control (A)	Experimental (B)	Cooperative (C)
Pre-test	**N**	20	20	20
	Mean	37.78%	38.44%	37.08%
	S. D.	14.82	16.18	15.36
Post-test	**N**	20	20	20
	Mean	39.78%	58.67%	73.84%
	S. D.	12.56	12.31	13.81
<g>	**Mean**	0.0321	0.3286	0.5842
	S. D.	0.102	0.131	0.146

To compare effects of different treatments on the three groups, the *t*-value and p-value were obtained, using average normalized gain and corresponding standard deviation of each group. The *t*-value between every pair of group was compared with $t_{critical}$ at the 0.01 level of significance as shown in the following Table 3

TABLE 3

Comparison of t-values and p-values in between the groups

	Groups A and B	**Groups B and C**	**Groups A and C**
t- value	7.986	6.885	13.86
p- value	1.19×10^{-9}	3.52×10^{-8}	1.92×10^{-16}

[For significance, $t_{critical}$ = 2.71 at 0.01 level for df = 38]

Table 2 shows that the average normalized gain for experimental group was found to be <g> = 0.3286 and that of control group was found to be <g> = 0.0321. The *t*-test was conducted on normalized gains of these groups. The difference between two normalized mean was significant at the 0.01 alpha level of significance (t =7.986). Since the average normalized gain for experimental group is greater than 0.3, the treatment used for experimental group is almost *ten times* effective. The average normalized gain shows that the CES supported teaching of calculus in the classroom is effective than the traditional teaching method in promoting conceptual understanding.

Table 2 shows that, the average normalized gain for experimental group was found to be <g> = 0.3286 and that of cooperative group was found to be <g> = 0.5842. The t-test was conducted on normalized gains of these groups. The difference between two normalized mean was significant at the 0.01 alpha level of significance (t =6.885). The average normalized gain for the cooperative group is *1.77 times* effective than the experimental group.

From Table 2, the average normalized gain for control group was found to be <g> = 0.0321 and that of cooperative group was found to be <g> = 0.5842. The t-test was conducted on normalized gains of these groups. The difference between two normalized mean was significant at the 0.01 alpha level of significance (t =13.86). The average normalized gain for the cooperative group is *eighteen times* effective than the control group.

Conclusions and Suggestions:

The finding of this study suggests that the instructional method used by researcher is effective in enhancing mathematical reasoning skills and conceptual understanding of calculus at undergraduate level. The study also indicates that computer aided instructions have greater potential to advance a conceptual change by helping students to move from their misconceptions to correct conceptions. But if additional support of cooperative learning using CES is provided to students then a large gain in conceptual understanding has been observed. From this study, researcher suggests that computer assisted learning with the aid of either suitable, effective mathematical educational software or self developed software (by analyzing misconceptions about the respective topic to end up difficulties in understanding and induce proper and clear concepts that result significantly), convey significant change in conceptual understanding of students. It was observed that the computer tool 'CES' intended to help students to learn and understand the concepts in calculus by facilitating enriched environments of interactive learning as well as found helpful tool for teachers to explain abstract concepts through powerful visualizations.

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